Precalculus

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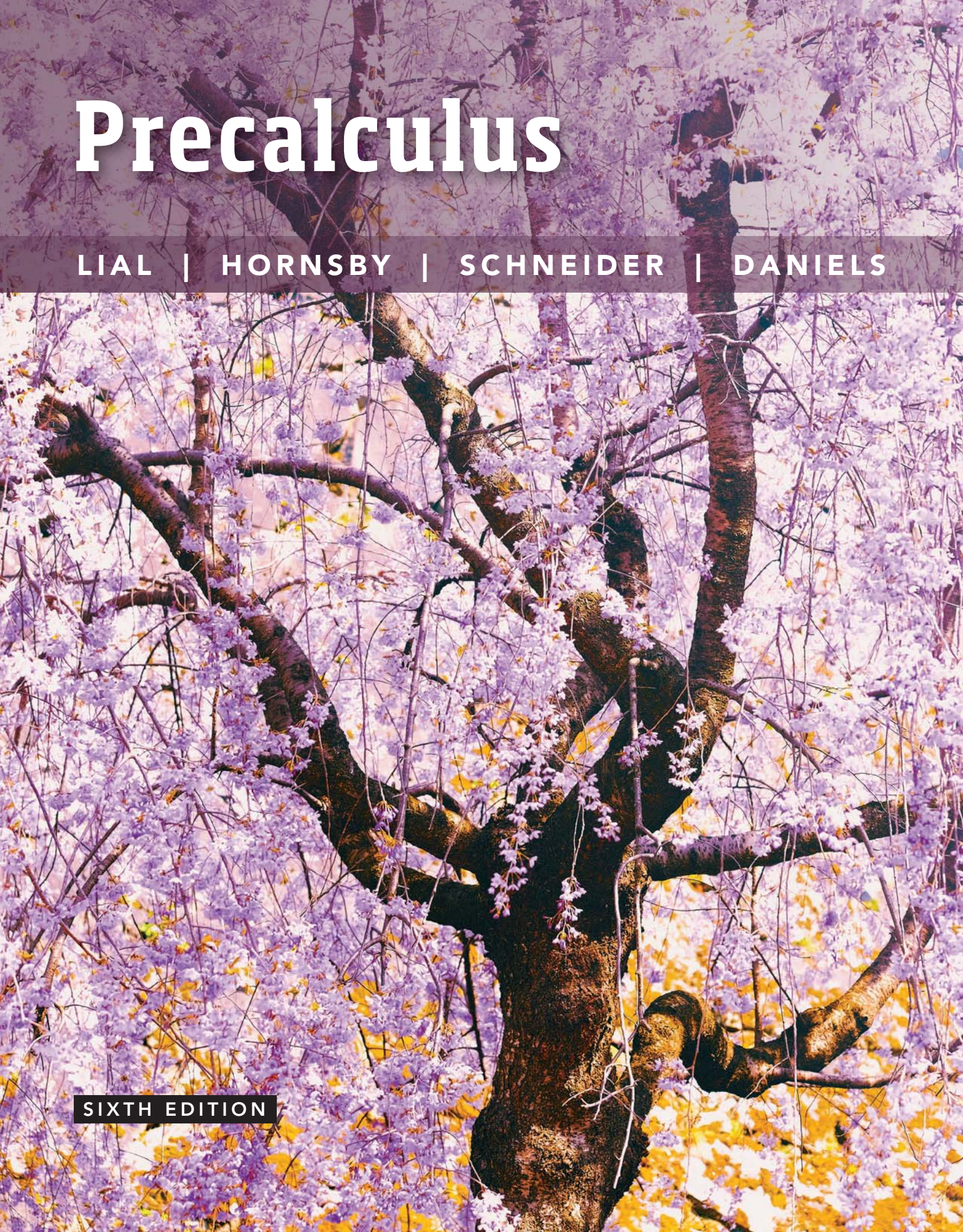

SIXTH EDITION

Precalculus

SIXTH EDITION

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Library of Congress Cataloging-in-Publication Data

Names: Lial, Margaret L., author.

Title: Precalculus / Margaret L. Lial, American River College [and three others]..

Description: 6th edition. | Boston : Pearson, [2017]

Identifiers: LCCN 2015045217 | ISBN 9780134217420 (hardcover) | ISBN 013421742X (hardcover) | ISBN 9780134217635 (hardcover) | ISBN 0134217632 (hardcover)

Subjects: LCSH: Precalculus. | Algebra. | Trigonometry.

Classification: LCC QA154.3 .L56 2017 | DDC 512--dc23

LC record available at http://lccn.loc.gov/2015045217

1 2 3 4 5 6 7 8 9 10—CRK—18 17 16 15

PEARSON

www.pearsonhighered.com

ISBN 13: 978-0-13-421742-0
ISBN 10: 0-13-421742-X

To Rhonda, Sandy, and Betty
Johnny

To my MS & T professors, Gus Garver, Troy Hicks, and Jagdish Patel
C.J.D.

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Preface

WELCOME TO THE 6TH EDITION

In the sixth edition of *Precalculus,* we continue our ongoing commitment to providing the best possible text to help instructors teach and students succeed. In this edition, we have remained true to the pedagogical style of the past while staying focused on the needs of today's students. Support for all classroom types (traditional, hybrid, and online) may be found in this classic text and its supplements backed by the power of Pearson's MyMathLab.

In this edition, we have drawn upon the extensive teaching experience of the Lial team, with special consideration given to reviewer suggestions. General updates include enhanced readability with improved layout of examples, better use of color in displays, and language written with students in mind. All calculator screenshots have been updated and now provide color displays to enhance students' conceptual understanding. Each homework section now begins with a group of *Concept Preview* exercises, assignable in MyMathLab, which may be used to ensure students' understanding of vocabulary and basic concepts prior to beginning the regular homework exercises.

Further enhancements include numerous current data examples and exercises that have been updated to reflect current information. Additional real-life exercises have been included to pique student interest; answers to writing exercises have been provided; better consistency has been achieved between the directions that introduce examples and those that introduce the corresponding exercises; and better guidance for rounding of answers has been provided in the exercise sets.

The Lial team believes this to be our best *Precalculus* edition yet, and we sincerely hope that you enjoy using it as much as we have enjoyed writing it. Additional textbooks in this series are

College Algebra, Twelfth Edition
Trigonometry, Eleventh Edition
College Algebra & Trigonometry, Sixth Edition.

HIGHLIGHTS OF NEW CONTENT

- In **Chapter R,** more detail has been added to set-builder notation, illustrations of the rules for exponents have been provided, and many exercises have been updated to better match section examples.
- Several new and updated application exercises have been inserted into the **Chapter 1** exercise sets. New objectives have been added to **Section 1.4** outlining the four methods for solving a quadratic equation, along with guidance suggesting when each method may be used efficiently.
- **Chapters 2 and 3** contain numerous new and updated application exercises, along with many updated calculator screenshots that are now provided in color. In response to reviewer suggestions, the discussion on increasing, decreasing, and constant functions in **Section 2.3** has been written to apply to open intervals of the domain. Also as a response to reviewers, intercepts of graphs are now defined in terms of coordinates rather than a single number. This notation continues throughout the text.
- In **Chapter 4,** greater emphasis is given to the concept of exponential and logarithmic functions as inverses, there is a new table providing descriptions of the additional properties of exponents, and additional exercises requiring graphing logarithmic functions with translations have been included. There are also many new and updated real-life applications of exponential and logarithmic functions.

- In **Chapter 5,** we now include historical material for students to see how trigonometry developed as a means to solve applied problems involving right triangles. In this chapter and the others that cover trigonometry, we have reorganized exercise sets to correspond to the flow of the examples when necessary.
- **Chapter 6** continues to focus on the periodic nature of the circular functions. To illustrate, we have added exercises that involve data of average monthly temperatures of regions that lie below the equator, as well as data that describe the fractional part of the moon illuminated for each day of a particular month. A new example (and corresponding exercises) for analyzing damped oscillatory motion has been included in **Section 6.7.**
- **Chapter 7** now includes a derivation of the product-to-sum identity for the product $\sin A \cos B$, as well as new figures illustrating periodic functions associated with music tones and frequencies.
- In **Chapter 8,** we have reorganized the two sections dealing with vectors. The material covered has not changed, but we have rewritten the sections so that **Section 8.3** first covers geometrically defined vectors and applications, while **Section 8.4** then introduces algebraically defined vectors and the dot product.
- In **Chapter 9,** special attention has been given to finding partial fraction decompositions in **Section 9.4** and to linear programing in **Section 9.6.** Examples have been rewritten to promote student understanding of these very difficult topics.
- In **Chapter 10,** greater emphasis is given to analyzing the specific aspects of conic sections, such as finding the equation of the axis of symmetry of a parabola, finding the coordinates of the foci of ellipses and hyperbolas, and finding the equations of the asymptotes of hyperbolas.
- Throughout **Chapter 11,** examples have been carefully updated to ensure that students are able to understand each step of the solutions. Special consideration was given to mathematical induction in **Section 11.5** by providing numerous additional side comments for the steps in the solution of examples in this difficult section.
- For visual learners, numbered **Figure** and **Example** references within the text are set using the same typeface as the figure number itself and bold print for the example. This makes it easier for the students to identify and connect them. We also have increased our use of a "drop down" style, when appropriate, to distinguish between simplifying expressions and solving equations, and we have added many more explanatory side comments. Guided Visualizations, with accompanying exercises and explorations, are now available and assignable in MyMathLab.
- *Precalculus* is widely recognized for the quality of its exercises. In the sixth edition, nearly 1500 are new or modified, and hundreds present updated real-life data. Furthermore, the MyMathLab course has expanded coverage of all exercise types appearing in the exercise sets, as well as the mid-chapter Quizzes and Summary Exercises.

FEATURES OF THIS TEXT

SUPPORT FOR LEARNING CONCEPTS

We provide a variety of features to support students' learning of the essential topics of precalculus. Explanations that are written in understandable terms, figures and graphs that illustrate examples and concepts, graphing technology that supports and

enhances algebraic manipulations, and real-life applications that enrich the topics with meaning all provide opportunities for students to deepen their understanding of mathematics. These features help students make mathematical connections and expand their own knowledge base.

- **Examples** Numbered examples that illustrate the techniques for working exercises are found in every section. We use traditional explanations, side comments, and pointers to describe the steps taken—and to warn students about common pitfalls. Some examples provide additional graphing calculator solutions, although these can be omitted if desired.
- **Now Try Exercises** Following each numbered example, the student is directed to try a corresponding odd-numbered exercise (or exercises). This feature allows for quick feedback to determine whether the student has understood the principles illustrated in the example.
- **Real-Life Applications** We have included hundreds of real-life applications, many with data updated from the previous edition. They come from fields such as business, entertainment, sports, biology, astronomy, geology, music, highway design, and environmental studies.
- **Function Boxes** Beginning in Chapter 2, functions provide a unifying theme throughout the text. Special function boxes offer a comprehensive, visual introduction to each type of function and also serve as an excellent resource for reference and review. Each function box includes a table of values, traditional and calculator-generated graphs, the domain, the range, and other special information about the function. These boxes are assignable in MyMathLab.
- **Figures and Photos** Today's students are more visually oriented than ever before, and we have updated the figures and photos in this edition to promote visual appeal. Guided Visualizations with accompanying exercises and explorations are now available and assignable in MyMathLab.
- **Use of Graphing Technology** We have integrated the use of graphing calculators where appropriate, although ***this technology is completely optional and can be omitted without loss of continuity.*** We continue to stress that graphing calculators support understanding but that students must first master the underlying mathematical concepts. Exercises that require the use of a graphing calculator are marked with the icon .
- **Cautions and Notes** Text that is marked **CAUTION** warns students of common errors, and **NOTE** comments point out explanations that should receive particular attention.
- **Looking Ahead to Calculus** These margin notes offer glimpses of how the topics currently being studied are used in calculus.

SUPPORT FOR PRACTICING CONCEPTS

This text offers a wide variety of exercises to help students master precalculus. The extensive exercise sets provide ample opportunity for practice, and the exercise problems increase in difficulty so that students at every level of understanding are challenged. The variety of exercise types promotes understanding of the concepts and reduces the need for rote memorization.

- **NEW Concept Preview** Each exercise set now begins with a group of **CONCEPT PREVIEW** exercises designed to promote understanding of vocabulary and basic concepts of each section. These new exercises are assignable in MyMathLab and will provide support especially for hybrid, online, and flipped courses.

- **Exercise Sets** In addition to traditional drill exercises, this text includes writing exercises, optional graphing calculator problems , and multiple-choice, matching, true/false, and completion exercises. Those marked ***Concept Check*** focus on conceptual thinking. ***Connecting Graphs with Equations*** exercises challenge students to write equations that correspond to given graphs.

- **Relating Concepts Exercises** Appearing at the end of selected exercise sets, these groups of exercises are designed so that students who work them in numerical order will follow a line of reasoning that leads to an understanding of how various topics and concepts are related. All answers to these exercises appear in the student answer section, and these exercises are assignable in MyMathLab.

- **Complete Solutions to Selected Exercises** Exercise numbers marked indicate that a full worked-out solution appears in the eText. These are often exercises that extend the skills and concepts presented in the numbered examples.

SUPPORT FOR REVIEW AND TEST PREP

Ample opportunities for review are found within the chapters and at the ends of chapters. Quizzes that are interspersed within chapters provide a quick assessment of students' understanding of the material presented up to that point in the chapter. Chapter "Test Preps" provide comprehensive study aids to help students prepare for tests.

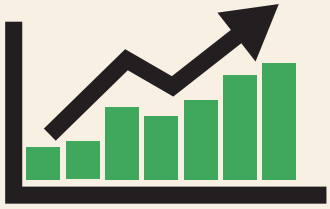
- **Quizzes** Students can periodically check their progress with in-chapter quizzes that appear in all chapters, beginning with Chapter 1. All answers, with corresponding section references, appear in the student answer section. These quizzes are assignable in MyMathLab.

- **Summary Exercises** These sets of in-chapter exercises give students the all-important opportunity to work *mixed* review exercises, requiring them to synthesize concepts and select appropriate solution methods. The summary exercises are assignable in MyMathLab.

- **End-of-Chapter Test Prep** Following the final numbered section in each chapter, the Test Prep provides a list of **Key Terms,** a list of **New Symbols** (if applicable), and a two-column **Quick Review** that includes a section-by-section summary of concepts and examples. This feature concludes with a comprehensive set of **Review Exercises** and a **Chapter Test.** The Test Prep, Review Exercises, and Chapter Test are assignable in MyMathLab.

BREAK THROUGH
To improving results

Get the most out of MyMathLab®



MyMathLab is the world's leading online resource for teaching and learning mathematics. MyMathLab helps students and instructors improve results, and it provides engaging experiences and personalized learning for each student so learning can happen in any environment. Plus, it offers flexible and time-saving course management features to allow instructors to easily manage their classes while remaining in complete control, regardless of course format.

Personalized Support for Students

- MyMathLab comes with many learning resources–eText, animations, videos, and more–all designed to support your students as they progress through their course.
- The Adaptive Study Plan acts as a personal tutor, updating in real time based on student performance to provide personalized recommendations on what to work on next. With the new Companion Study Plan assignments, instructors can now assign the Study Plan as a prerequisite to a test or quiz, helping to guide students through concepts they need to master.
- Personalized Homework enables instructors to create homework assignments tailored to each student's specific needs and focused on the topics they have not yet mastered.

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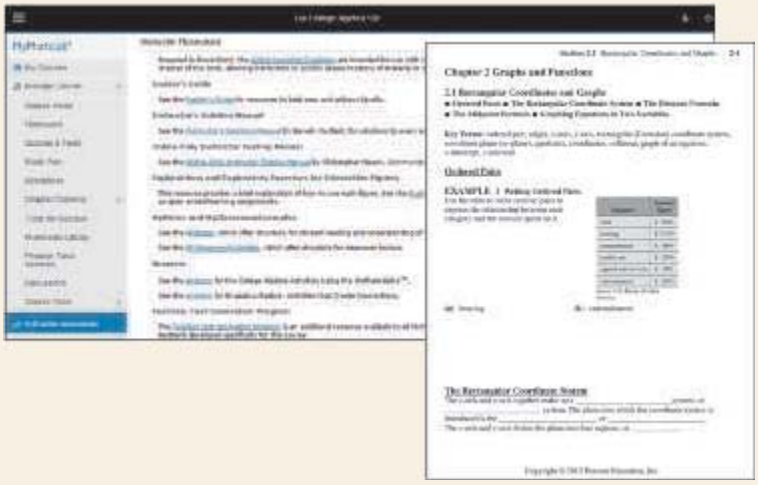

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ACKNOWLEDGMENTS

We wish to thank the following individuals who provided valuable input into this edition of the text.

Barbara Aramenta – Pima Community College
Robert Bates – Honolulu Community College
Troy Brachey – Tennessee Tech University
Hugh Cornell – University of North Florida
John E. Daniels – Central Michigan University
Dan Fahringer – HACC Harrisburg
Doug Grenier – Rogers State University
Mary Hill – College of DuPage
Keith Hubbard – Stephen Austin State University
Christine Janowiak – Arapahoe Community College
Tarcia Jones – Austin Community College, Rio Grande
Rene Lumampao – Austin Community College
Rosana Maldonado – South Texas Community College, Pecan
Nilay S. Manzagol – Georgia State University
Marianna McClymonds – Phoenix College
Randy Nichols – Delta College
Preeti Parikh – SUNY Maritime
Deanna Robinson-Briedel – McLennan Community College
Sutandra Sarkar – Georgia State University
Patty Schovanec – Texas Tech University
Jimmy Vincente – El Centro College
Deanna M. Welsch – Illinois Central College
Amanda Wheeler – Amarillo College
Li Zhou – Polk State College

Our sincere thanks to those individuals at Pearson Education who have supported us throughout this revision: Anne Kelly, Christine O'Brien, Joe Vetere, and Danielle Simbajon. Terry McGinnis continues to provide behind-the-scenes guidance for both content and production. We have come to rely on her expertise during all phases of the revision process. Marilyn Dwyer of Cenveo® Publishing Services, with the assistance of Carol Merrigan, provided excellent production work. Special thanks go out to Paul Lorczak, John Morin, and Perian Herring for their excellent accuracy-checking. We thank Lucie Haskins, who provided an accurate index, and Jack Hornsby, who provided assistance in creating calculator screens, researching data updates, and proofreading. We appreciate the valuable suggestions for Chapter 9 that Mary Hill of *College of Dupage* made during our meeting with her in March 2010.

As an author team, we are committed to providing the best possible precalculus course to help instructors teach and students succeed. As we continue to work toward this goal, we welcome any comments or suggestions you might send, via e-mail, to math@pearson.com.

Margaret L. Lial
John Hornsby
David I. Schneider
Callie J. Daniels

R Review of Basic Concepts

Positive and negative numbers, used to represent gains and losses on a board such as this one, are examples of *real numbers* encountered in applications of mathematics.

- **R.1** Sets
- **R.2** Real Numbers and Their Properties
- **R.3** Polynomials
- **R.4** Factoring Polynomials
- **R.5** Rational Expressions
- **R.6** Rational Exponents
- **R.7** Radical Expressions

R.1 Sets

- Basic Definitions
- Operations on Sets

Basic Definitions

A **set** is a collection of objects. The objects that belong to a set are its **elements,** or **members.** In algebra, the elements of a set are usually numbers. Sets are commonly written using **set braces,** { }.

$\{1, 2, 3, 4\}$ The set containing the elements 1, 2, 3, and 4

The order in which the elements are listed is not important. As a result, this same set can also be written as $\{4, 3, 2, 1\}$ or with any other arrangement of the four numbers.

To show that 4 is an element of the set $\{1, 2, 3, 4\}$, we use the symbol $\in$.

$$4 \in \{1, 2, 3, 4\}$$

Since 5 is *not* an element of this set, we place a slash through the symbol $\in$.

$$5 \notin \{1, 2, 3, 4\}$$

It is customary to name sets with capital letters.

$S = \{1, 2, 3, 4\}$ S is used to name the set.

Set S was written above by listing its elements. Set S might also be described as

"the set containing the first four counting numbers."

The set F, consisting of all fractions between 0 and 1, is an example of an **infinite set**—one that has an unending list of distinct elements. A **finite set** is one that has a limited number of elements. The process of counting its elements comes to an end.

Some infinite sets can be described by listing. For example, the set of numbers N used for counting, which are the **natural numbers** or the **counting numbers,** can be written as follows.

$$N = \{1, 2, 3, 4, \ldots\}$$ Natural (counting) numbers

The three dots (*ellipsis points*) show that the list of elements of the set continues according to the established pattern.

Sets are often written in **set-builder notation,** which uses a variable, such as x, to describe the elements of the set. The following set-builder notation represents the set $\{3, 4, 5, 6\}$ and is read "the set of all elements x such that x is a natural number between 2 and 7." The numbers 2 and 7 are *not* between 2 and 7.

$\{x \mid x \text{ is a natural number between 2 and 7}\} = \{3, 4, 5, 6\}$ Set-builder notation

The set of all elements x — such that — x is a natural number between 2 and 7

EXAMPLE 1 Using Set Notation and Terminology

Identify each set as *finite* or *infinite*. Then determine whether 10 is an element of the set.

(a) $\{7, 8, 9, \ldots, 14\}$

(b) $\left\{1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots\right\}$

(c) $\{x \mid x \text{ is a fraction between 1 and 2}\}$

(d) $\{x \mid x \text{ is a natural number between 9 and 11}\}$

SOLUTION

(a) The set is finite, because the process of counting its elements 7, 8, 9, 10, 11, 12, 13, and 14 comes to an end. The number 10 belongs to the set.

$$10 \in \{7, 8, 9, \ldots, 14\}$$

(b) The set is infinite, because the ellipsis points indicate that the pattern continues indefinitely. In this case,

$$10 \notin \left\{1, \tfrac{1}{4}, \tfrac{1}{16}, \tfrac{1}{64}, \ldots\right\}.$$

(c) Between any two distinct natural numbers there are infinitely many fractions, so this set is infinite. The number 10 is not an element.

(d) There is only one natural number between 9 and 11, namely 10. So the set is finite, and 10 is an element.

✔ **Now Try Exercises 11, 13, 15, and 17.**

EXAMPLE 2 Listing the Elements of a Set

Use set notation, and list all the elements of each set.

(a) $\{x \mid x \text{ is a natural number less than } 5\}$

(b) $\{x \mid x \text{ is a natural number greater than 7 and less than } 14\}$

SOLUTION

(a) The natural numbers less than 5 form the set $\{1, 2, 3, 4\}$.

(b) This is the set $\{8, 9, 10, 11, 12, 13\}$. ✔ **Now Try Exercise 25.**

When we are discussing a particular situation or problem, the **universal set** (whether expressed or implied) contains all the elements included in the discussion. The letter U is used to represent the universal set. The **null set,** or **empty set,** is the set containing no elements. We write the null set by either using the special symbol $\emptyset$, or else writing set braces enclosing no elements, $\{\ \}$.

CAUTION Do not combine these symbols. $\{\emptyset\}$ ***is not the null set.*** It is the set containing the symbol $\emptyset$.

Every element of the set $S = \{1, 2, 3, 4\}$ is a natural number. S is an example of a *subset* of the set N of natural numbers. This relationship is written using the symbol $\subseteq$.

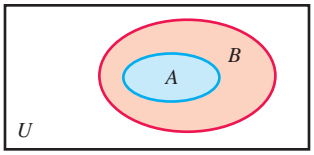
$$S \subseteq N$$

By definition, set A is a **subset** of set B if every element of set A is also an element of set B. For example, if $A = \{2, 5, 9\}$ and $B = \{2, 3, 5, 6, 9, 10\}$, then $A \subseteq B$. However, there are some elements of B that are not in A, so B is not a subset of A. This relationship is written using the symbol $\not\subseteq$.

$$B \not\subseteq A$$

Every set is a subset of itself. Also, $\emptyset$ is a subset of every set.

If A is any set, then $A \subseteq A$ and $\emptyset \subseteq A$.



$A \subseteq B$

Figure 1

Figure 1 shows a set A that is a subset of set B. The rectangle in the drawing represents the universal set U. Such a diagram is a **Venn diagram.**

Two sets A and B are equal whenever $A \subseteq B$ and $B \subseteq A$. Equivalently, $A = B$ if the two sets contain exactly the same elements. For example,

$$\{1, 2, 3\} = \{3, 1, 2\}$$

is true because both sets contain exactly the same elements. However,

$$\{1, 2, 3\} \neq \{0, 1, 2, 3\}$$

because the set $\{0, 1, 2, 3\}$ contains the element 0, which is not an element of $\{1, 2, 3\}$.

EXAMPLE 3 Examining Subset Relationships

Let $U = \{1, 3, 5, 7, 9, 11, 13\}$, $A = \{1, 3, 5, 7, 9, 11\}$, $B = \{1, 3, 7, 9\}$, $C = \{3, 9, 11\}$, and $D = \{1, 9\}$. Determine whether each statement is *true* or *false*.

(a) $D \subseteq B$ **(b)** $B \subseteq D$ **(c)** $C \not\subseteq A$ **(d)** $U = A$

SOLUTION

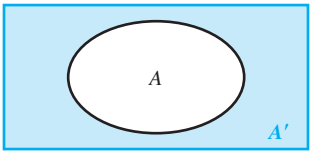
(a) All elements of D, namely 1 and 9, are also elements of B, so D is a subset of B, and $D \subseteq B$ is true.

(b) There is at least one element of B (for example, 3) that is not an element of D, so B is *not* a subset of D. Thus, $B \subseteq D$ is false.

(c) C is a subset of A, because every element of C is also an element of A. Thus, $C \subseteq A$ is true, and as a result, $C \not\subseteq A$ is false.

(d) U contains the element 13, but A does not. Therefore, $U = A$ is false.

Now Try Exercises 53, 55, 63, and 65.



A'

Figure 2

Operations on Sets Given a set A and a universal set U, the set of all elements of U that do ***not*** belong to set A is the **complement** of set A. For example, if set A is the set of all students in a class 30 years old or older, and set U is the set of all students in the class, then the complement of A would be the set of all students in the class younger than age 30.

The complement of set A is written A' (read **"A-prime"**). The Venn diagram in **Figure 2** shows a set A. Its complement, A', is in color. Using set-builder notation, the complement of set A is described as follows.

$$A' = \{x | x \in U, \quad x \notin A\}$$

EXAMPLE 4 Finding Complements of Sets

Let $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 3, 5, 7\}$, and $B = \{3, 4, 6\}$. Find each set.

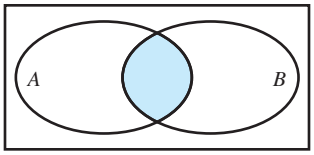
(a) A' **(b)** B' **(c)** $\varnothing'$ **(d)** U'

SOLUTION

(a) Set A' contains the elements of U that are not in A. Thus, $A' = \{2, 4, 6\}$.

(b) $B' = \{1, 2, 5, 7\}$ **(c)** $\varnothing' = U$ **(d)** $U' = \varnothing$

Now Try Exercise 89.



$A \cap B$

Figure 3

Given two sets A and B, the set of all elements belonging both to set A ***and*** to set B is the **intersection** of the two sets, written $A \cap B$. For example, if $A = \{1, 2, 4, 5, 7\}$ and $B = \{2, 4, 5, 7, 9, 11\}$, then we have the following.

$$A \cap B = \{1, 2, 4, 5, 7\} \cap \{2, 4, 5, 7, 9, 11\} = \{2, 4, 5, 7\}$$

The Venn diagram in **Figure 3** shows two sets A and B. Their intersection, $A \cap B$, is in color. Using set-builder notation, the intersection of sets A and B is described as follows.

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

Two sets that have no elements in common are **disjoint sets.** If A and B are any two disjoint sets, then $A \cap B = \emptyset$. For example, there are no elements common to both $\{50, 51, 54\}$ and $\{52, 53, 55, 56\}$, so these two sets are disjoint.

$$\{50, 51, 54\} \cap \{52, 53, 55, 56\} = \emptyset$$

EXAMPLE 5 **Finding Intersections of Two Sets**

Find each of the following. Identify any disjoint sets.

(a) $\{9, 15, 25, 36\} \cap \{15, 20, 25, 30, 35\}$

(b) $\{2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4\}$

(c) $\{1, 3, 5\} \cap \{2, 4, 6\}$

SOLUTION

(a) $\{9, 15, 25, 36\} \cap \{15, 20, 25, 30, 35\} = \{15, 25\}$

The elements 15 and 25 are the only ones belonging to both sets.

(b) $\{2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4\} = \{2, 3, 4\}$

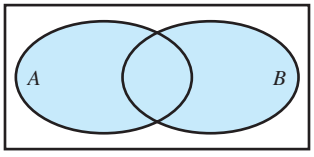
(c) $\{1, 3, 5\} \cap \{2, 4, 6\} = \emptyset$ Disjoint sets

Now Try Exercises 69, 75, and 85.

The set of all elements belonging to set A ***or*** to set B (or to both) is the **union** of the two sets, written $A \cup B$. For example, if $A = \{1, 3, 5\}$ and $B = \{3, 5, 7, 9\}$, then we have the following.

$$A \cup B = \{1, 3, 5\} \cup \{3, 5, 7, 9\} = \{1, 3, 5, 7, 9\}$$

The Venn diagram in **Figure 4** shows two sets A and B. Their union, $A \cup B$, is in color.



$A \cup B$

Figure 4

Using set-builder notation, the union of sets A and B is described as follows.

$$A \cup B = \{x | x \in A \text{ or } x \in B\}$$

EXAMPLE 6 Finding Unions of Two Sets

Find each of the following.

(a) $\{1, 2, 5, 9, 14\} \cup \{1, 3, 4, 8\}$

(b) $\{1, 3, 5, 7\} \cup \{2, 4, 6\}$

(c) $\{1, 3, 5, 7, \ldots\} \cup \{2, 4, 6, \ldots\}$

SOLUTION

(a) Begin by listing the elements of the first set, $\{1, 2, 5, 9, 14\}$. Then include any elements from the second set that are not already listed.

$$\{1, 2, 5, 9, 14\} \cup \{1, 3, 4, 8\} = \{1, 2, 3, 4, 5, 8, 9, 14\}$$

(b) $\{1, 3, 5, 7\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 5, 6, 7\}$

(c) $\{1, 3, 5, 7, \ldots\} \cup \{2, 4, 6, \ldots\} = N$ Natural numbers

Now Try Exercises 71 and 83.

The **set operations** are summarized below.

Set Operations

Let A and B define sets, with universal set U.

The **complement** of set A is the set A' of all elements in the universal set that do *not* belong to set A.

$$A' = \{x | x \in U, \quad x \notin A\}$$

The **intersection** of sets A and B, written $A \cap B$, is made up of all the elements belonging to both set A *and* set B.

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

The **union** of sets A and B, written $A \cup B$, is made up of all the elements belonging to set A *or* set B.

$$A \cup B = \{x | x \in A \text{ or } x \in B\}$$

R.1 Exercises

CONCEPT PREVIEW *Fill in the blank to correctly complete each sentence.*

1. The elements of the set of natural numbers are _____.
2. Set A is a(n) _____ of set B if every element of set A is also an element of set B.
3. The set of all elements of the universal set U that do not belong to set A is the _____ of set A.
4. The _____ of sets A and B is made up of all the elements belonging to both set A *and* set B.
5. The _____ of sets A and B is made up of all the elements belonging to set A *or* set B (or both).

CONCEPT PREVIEW *Work each problem.*

6. Identify the set $\left\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\right\}$ as finite or infinite.

7. Use set notation and write the elements belonging to the set $\{x \mid x$ is a natural number less than $6\}$.

8. Let $U = \{1, 2, 3, 4, 5\}$ and $A = \{1, 2, 3\}$. Find A'.

9. Find $\{16, 18, 21, 50\} \cap \{15, 16, 17, 18\}$.

10. Find $\{16, 18, 21, 50\} \cup \{15, 16, 17, 18\}$.

Identify each set as finite *or* infinite. *Then determine whether* 10 *is an element of the set.* ***See Example 1.***

11. $\{4, 5, 6, \ldots, 15\}$

12. $\{1, 2, 3, 4, 5, \ldots, 75\}$

13. $\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\right\}$

14. $\{4, 5, 6, \ldots\}$

15. $\{x \mid x$ is a natural number greater than $11\}$

16. $\{x \mid x$ is a natural number greater than or equal to $10\}$

17. $\{x \mid x$ is a fraction between 1 and $2\}$

18. $\{x \mid x$ is an even natural number$\}$

Use set notation, and list all the elements of each set. ***See Example 2.***

19. $\{12, 13, 14, \ldots, 20\}$

20. $\{8, 9, 10, \ldots, 17\}$

21. $\left\{1, \frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{32}\right\}$

22. $\{3, 9, 27, \ldots, 729\}$

23. $\{17, 22, 27, \ldots, 47\}$

24. $\{74, 68, 62, \ldots, 38\}$

25. $\{x \mid x$ is a natural number greater than 8 and less than $15\}$

26. $\{x \mid x$ is a natural number not greater than $4\}$

Insert $\in$ *or* $\notin$ *in each blank to make the resulting statement true.* ***See Examples 1 and 2.***

27. $6 ____ \{3, 4, 5, 6\}$

28. $9 ____ \{2, 3, 5, 9, 8\}$

29. $5 ____ \{4, 6, 8, 10\}$

30. $13 ____ \{3, 5, 12, 14\}$

31. $0 ____ \{0, 2, 3, 4\}$

32. $0 ____ \{0, 5, 6, 7, 8, 10\}$

33. $\{3\} ____ \{2, 3, 4, 5\}$

34. $\{5\} ____ \{3, 4, 5, 6, 7\}$

35. $\{0\} ____ \{0, 1, 2, 5\}$

36. $\{2\} ____ \{2, 4, 6, 8\}$

37. $0 ____ \varnothing$

38. $\varnothing ____ \varnothing$

Determine whether each statement is true *or* false. ***See Examples 1–3.***

39. $3 \in \{2, 5, 6, 8\}$

40. $6 \in \{2, 5, 8, 9\}$

41. $1 \in \{11, 5, 4, 3, 1\}$

42. $12 \in \{18, 17, 15, 13, 12\}$

43. $9 \notin \{8, 5, 2, 1\}$

44. $3 \notin \{7, 6, 5, 4\}$

45. $\{2, 5, 8, 9\} = \{2, 5, 9, 8\}$

46. $\{3, 0, 9, 6, 2\} = \{2, 9, 0, 3, 6\}$

47. $\{5, 8, 9\} = \{5, 8, 9, 0\}$

48. $\{3, 7, 12, 14\} = \{3, 7, 12, 14, 0\}$

49. $\{x \mid x$ is a natural number less than $3\} = \{1, 2\}$

50. $\{x \mid x$ is a natural number greater than $10\} = \{11, 12, 13, \ldots\}$